Quiz 9
Chemical Engineering Thermodynamics
April 2, 2015

1)
P8.7 Ethylene at 350°C and 50 bar is passed through an adiabatic expander to obtain work and exits at 2 bar. If the expander has an efficiency of 80%, how much work is obtained per mole of ethylene, and what is the final temperature of the ethylene? How does the final temperature compare with what would be expected from a reversible expander?

See equations below

T (K)	623.15	Z	V	H	U	S
P (MPa)	5		cm3/gmol	J/mol	J/mol	J/molK
& for 1 root region		0.986361	1022.041	18750.12	13639.92	**9.1954**

T (K)	**404.71**	Z	V	H	U	S
P (MPa)	0.2		cm3/gmol	J/mol	J/mol	J/molK
& for 1 root region		0.99499	16739.56	5234.912	1887	**9.1954**

T (K)	452.012672	fugacity	H	U	S
P (MPa)	0.2	MPa	J/mol	J/mol	J/molK
answers for three root region		#NUM!	#NUM!	#NUM!	#NUM!
		#NUM!	#NUM!	#NUM!	#NUM!
& for 1 root region		0.199324	7937.95	4192.62	15.50748631

2)
8.10 Derive the integrated formula for the Helmholtz energy departure for the virial equation (Eqn. 7.7), where *B* is dependent on temperature only. Express your answer in terms of *B* and its temperature derivative.

Equation 7.7 given below with other useful expressions.

3)
9.1 The heat of fusion for the ice-water phase transition is 335 kJ/kg at 0 °C and 1 bar. The density of water is 1g/cm^3 at these conditions and that of ice is 0.915 g/cm^3. Develop an expression for the change of the melting temperature of ice as a function of pressure. Quantitatively explain why ice skates slide along the surface of ice for a 100 kg hockey player wearing 10 cm x 01 cm blades. Can it get too cold to ice skate? Would it be possible to ice skate on other materials such as solid CO_2?

See equations below.

$$\frac{(S-S^{ig})}{R} = \int_0^{\rho}\left[-T\left[\frac{\partial Z}{\partial T}\right]-(Z-1)\right]\frac{d\rho}{\rho}+\ln Z \tag{8.23}$$

$$\frac{(H-H^{ig})}{RT} = \int_0^{\rho}-T\left[\frac{\partial Z}{\partial T}\right]_{\rho}\frac{d\rho}{\rho}+Z-1 \tag{8.24}$$

$$\frac{(A-A^{ig})}{RT} = \int_0^{\rho}\frac{(Z-1)}{\rho}d\rho-\ln Z \tag{8.25}$$

$$\frac{(G-G^{ig})}{RT} = \int_0^{\rho}\frac{(Z-1)}{\rho}d\rho+(Z-1)-\ln Z \tag{8.26}$$

Useful formulas at fixed T,V include:

$$\frac{(A-A^{ig})_{TV}}{RT} = \int_0^{\rho}\frac{(Z-1)}{\rho}d\rho \tag{8.27}$$

$$\frac{(S-S^{ig})_{TV}}{R} = \int_0^{\rho}\left[-T\left[\frac{\partial Z}{\partial T}\right]_{\rho}-(Z-1)\right]\frac{d\rho}{\rho} \tag{8.28}$$

$$\left(\frac{H-H^{ig}}{RT}\right) = -\int_0^{P}T\left(\frac{\partial Z}{\partial T}\right)_P\frac{dP}{P} \tag{8.29}$$

$$\left(\frac{S-S^{ig}}{R}\right) = -\int_0^{P}\left[(Z-1)+T\left(\frac{\partial Z}{\partial T}\right)_P\right]\frac{dP}{P} \tag{8.30}$$

$$A = U - TS = H - PV - TS$$

$$\frac{(A-A^{ig})}{RT}=\frac{(H-H^{ig})}{RT}-\frac{(S-S^{ig})}{R}-Z+1$$

$$\frac{dP^{sat}}{dT} = \frac{\Delta H^{vap}}{T(V^V - V^L)}$$ 9.6

❶ Clapeyron equation.

$$S^V - S^L = \Delta S^{vap} = \frac{(H^V - H^L)}{T} = \frac{\Delta H^{vap}}{T}$$ 9.5

$$d\ln P^{sat} = \frac{-\Delta H^{vap}}{R(Z^V - Z^L)} d\left(\frac{1}{T}\right)$$ 9.7

❶ Clausius-Clapeyron equation.

$$d\ln P^{sat} = \frac{-\Delta H^{vap}}{R} d\left(\frac{1}{T}\right)$$ (ig) 9.8

$Z = 1 + (B^0 + \omega B^1)P_r/T_r$ or $Z = 1 + BP/RT$	7.6
where $B(T) = (B^0 + \omega B^1)RT_c/P_c$	7.7
$B^0 = 0.083 - 0.422/T_r^{1.6}$	7.8
$B^1 = 0.139 - 0.172/T_r^{4.2}$	7.9
Subject to $T_r > 0.686 + 0.439P_r$ or $V_r > 2.0$	7.10

R = 8.314 MPa cm³/(mole K°)

Answers Quiz 9
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1)
P8.7 Ethylene at 350°C and 50 bar is passed through an adiabatic expander to obtain work and exits at 2 bar. If the expander has an efficiency of 80%, how much work is obtained per mole of ethylene, and what is the final temperature of the ethylene? How does the final temperature compare with what would be expected from a reversible expander?

See equations below and PREOS.xlsx outputs.

(P8.7)

$$T_1 = 623.15K$$

$$P_1 = 5MPa$$

Use PREOS.XLS

$S_1 = S_2'$

T (K) 623.15	Z	V	H	U	S
P (MPa) 5		cm3/gmol	J/mol	J/mol	J/molK
& for 1 root region	0.986361	1022.041	18750.12	13639.92	**9.1954**

$\Rightarrow$ Use Solver in the spread sheet by changing pressure to 0.2MPa and Fixing the entropy value = 9.1954*J/mol-K* and in this case $S_1 = S_2'$=9.1954*J/mole-K* then we can find $\Delta H' and \Delta H$

T (K) **404.71**	Z	V	H	U	S
P (MPa) 0.2		cm3/gmol	J/mol	J/mol	J/molK
& for 1 root region	0.99499	16739.56	5234.912	1887	**9.1954**

$$\Rightarrow T = 404.71K$$

$$\Delta H' = 5234.912 - 18750.12 = -13515.21 J/mole$$

$$\eta = 0.8 \Rightarrow \frac{\Delta H}{\Delta H'} = 0.8 = \frac{W_s}{W_s'} = \frac{H_2 - H_1}{H_2' - H_1}$$

$$\Rightarrow 0.8 = \frac{H_2 - 18750.12}{5234.912 - 18750.12}$$

$$\Rightarrow H_2 = 7937.95 J/mol$$

$$\Rightarrow \Delta H = H_2 - H_1 = 7937.95 - 18750.12$$

$$\Rightarrow \Delta H = 10812.166 J/mol$$

T (K) 452.012672	fugacity	H	U	S
P (MPa) 0.2	MPa	J/mol	J/mol	J/molK
answers for three root region	#NUM!	#NUM!	#NUM!	#NUM!
	#NUM!	#NUM!	#NUM!	#NUM!
& for 1 root region	0.199324	7937.95	4192.62	15.50748631

$$\Rightarrow T_2 = 452K$$

2)

8.10 Derive the integrated formula for the Helmholtz energy departure for the virial equation (Eqn. 7.7), where B is dependent on temperature only. Express your answer in terms of B and its temperature derivative.

Equation 7.7 given below with other useful expressions.

(8.10) Derive the integrated formula for the Helmholtz energy departure ...

Solution:

$A = U - TS = H - PV - TS$

$$\frac{(A-A^{ig})}{RT} = \frac{(H-H^{ig})}{RT} - \frac{(S-S^{ig})}{R} - Z + 1$$

Using the form for $Z = Z(T,P)$: $\int_0^P \frac{(Z-1)}{P} dP - Z + 1 = \frac{BP}{RT} - \frac{BP}{RT} = 0$

3)

9.1 The heat of fusion for the ice-water phase transition is 335 kJ/kg at 0°C and 1 bar. The density of water is 1g/cm^3 at these conditions and that of ice is 0.915 g/cm^3. Develop an expression for the change of the melting temperature of ice as a function of pressure. Quantitatively explain why ice skates slide along the surface of ice for a 100 kg hockey player wearing 10 cm x 01 cm blades. Can it get too cold to ice skate? Would it be possible to ice skate on other materials such as solid CO_2?

(9.01) The heat of fusion for the ice-water phase transition is 335kJ/kg at 0°C

SOLUTION:

H_L-H_S =335 J/g (must add heat to melt ice)

$$V_L - V_s = \frac{1}{1g/cm^3} - \frac{1}{0.195g/cm^3} = -0.0929g/cm^3$$

$$\frac{dP^{sat}}{dT} = \frac{H_L - H_v}{T(V_L - V_v)} = \frac{335J/g}{(-0.0929)g/cm^3 * T[K]} = \frac{-3606(MPa)}{T(K)}$$

Assume ΔH and ΔV are constant with respect to P (i.e. ice and water are incompressible)

$$\Rightarrow \frac{dT}{T} = -\frac{dP}{3606[MPa]} \Rightarrow \frac{T_2}{T_1} = \exp\left(\frac{-(P_2 - P_1)}{3606[MPa]}\right)$$

For a 100 kg person with a 10 cm × 0.01cm blade,

$$\Rightarrow P = 9.8[m/s^2] \times 100[kg]/[0.1[m] \times 0.0001[m]] = 980 \times 10^5 Pa = 98MPa$$

$$\Rightarrow T_2^{sat} = 273 \times \left[\exp\left(\frac{-98}{3606}\right)\right] = -266K = -7^\circ C$$

But if the temperature is lower than -7°C, this person can no longer skate, and might need a sharper edge on the skates.

If replace ice with CO2 nobody can skate on it since $\Delta \hat{V} = \hat{V}_L - \hat{V}_S > 0$